CP Violation from Dimensional Reduction: Examples in 4+1 Dimensions.

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Abstract

We provide simple examples of the generation of complex mass terms and hence CP violation through dimensional reduction.

1 Introduction.

We regard the CP symmetry as fundamental. It is indeed the natural symmetry of gauge interactions, where it can be traced directly to the unitary nature of the gauge groups. A "pure" gauge theory (that is, without scalar interactions, such as Yukawa terms or even masses) is indeed CP-invariant in $3+1$ dimensions. The only possible source of violation stems from the anomalies (the so-called θ term) which can be rotated away for massless fermions. In the Standard Model CP violation thus comes only from the completely arbitrary Yukawa couplings.

The quest for unification would eventually eliminate the need for these independent coupling parameters, possibly relating them to the gauge interactions. It is thus to be expected that a unified theory should be CP-invariant. In such a case, a breaking mechanism is needed. It could be spontaneous (non-alignement of condensates or scalar vev), but the possibility also exists to relate it to some dynamical effect, for instance here, to dimensional reduction [1],[2].

In section 2, we will briefly discuss the discrete symmetries in $(d-1)+1$ dimensions, then construct in section 3 an explicit example. This example, while providing the equivalent of an electric dipole moment still has a rather fundamental problem, namely that the theory stays roughly vectorlike. Ways out are presented in section 4.

2 P, CP and CPT in $(d-1)+1$.

While this issue has been extensively tackled before, we recapitulate here a few salient points, and try to dissipate an apparent paradox [3].

There is a possible ambiguity in the definition of P. In 3 spatial dimensions, two definitions, namely the central inversion $\overrightarrow{x} \to -\overrightarrow{x}$ and the specular reflexion, say $x_1 \to -x_1$, are equivalent modulo one spatial rotation. For $(d-1)$ even however, the specular reflexion stays a discrete symmetry, while the central inversion is simply an element of the rotation group (with $det = -1$ and $+1$ respectively). Which is the best generalisation?

It turns out that the specular reflexion leads to the generalisation of the CPT theorem, which is a strong reason to choose it. In all dimensions, P may thus now be identified to $x_{d-1} \to -x_{d-1}$, or equivalently $x_1 \to -x_1$.

Another well-known statement is that the coupling $\bar{\psi}\psi$ in $4+1$ dimensions is P-violating. This may seem paradoxical. Indeed a scalar term in $3+1$ dimensions can be viewed as P-conserving, and we have just seen that P can be defined in a universal way whatever the number of dimensions. A clarification may be found in the more usual $3+1$ situation. Here indeed, we can have both

$$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$$

and

$$\bar{\psi}i\gamma_5\psi = i(\bar{\psi}_L\psi_R - \bar{\psi}_R\psi_L).$$

It is easy in $3+1$ dimensions to go from one type of coupling to the other by a mere sign flip of the semi-spinor (say $\psi_L \to -i\psi_L$). In fact, to achieve P violation in $3+1$ dimenions through spin-0 couplings, the simultaneous presence of $\bar{\psi}\psi$ and $\bar{\psi}\gamma_5\psi$ terms is needed. In $4+1$ dimensions, the $\bar{\psi}\gamma_4\psi$ component of the vector is automatically present

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in the kinetic part, and precisely corresponds to $i\bar{\psi}\gamma_5\psi$ in $3+1$ language through $\gamma_4 = i\gamma_5$. This in fact "locks" the definition and results in $\bar{\psi}\psi$ to be P-violating.

As we will see below, this is also at the origin of CP violation in the dimensional reduction process. With the definition of CP given above, the term $M\bar{\psi}\psi$ is easily seen to preserve CP in $4+1$ dimensions while breaking both C and P. For easy reference, we give below one possible representation of the C and P operators in $4+1$ dimensions:

$$C^{-1}_{(4+1)}\gamma_A C_{(4+1)} = \gamma_A^t \longrightarrow C = \gamma^1\gamma^3 = \gamma^2\gamma^0\gamma^5,$$
$$\psi^{P_{4+1}}(x_0, x_\mu, x_4) = \gamma^4\psi(x_0, x_\mu, -x_4),$$

$(A, B = 0, 1, 2, 3, 4)$.

3 First Examples.

We work for the moment in $4+1$ dimensions (the extension to $2n+1$ dimensions is easy) without specifying yet the nature of the extra spatial dimension (orbifold, compact or just infinite). Starting from $U(1)$ gauged Lagrangian for the fermions:

$$\mathcal{L} = i\bar{\psi}\not{D}\psi - M\,\bar{\psi}\psi,$$

with $D_B = \partial_B - ieA_B$. We observe immediately that breaking of $4+1$ in $3+1$ will introduce effective complex mass terms into the Dirac equation via possibly non-vanishing contribution arising from ∂_4 or A_4 terms, which we denote generally by X_4, resulting in a mass structure:

$$\bar{\psi}(M + i\gamma_5 X_4)\psi.$$

Such a structure could lead to CP violation (for instance, if a strong anomaly is also present, or, as was studied by Thirring [1], in the case of a non-minimal coupling of the photon). In the case of a pure minimal-coupling U(1) theory, the complex mass term can however be rotated away by a chiral rotation in $3+1$ dimensions, and CP violation thus requires at least an extension of the gauge group.

Let us for now concentrate on the possible origins of the contribution X_4.

The case considered by Thirring is the simplest; if the 4th dimension is compactified, and X_4 simply corresponds to the Kaluza-Klein mass $\frac{n}{R}$. The result is a tower of states, with no CP-violating effect for the fundamental.

If we want to separate the CP violation from the use of the excited Kaluza-Klein states, or use a different dimensional reduction scheme, the obvious solution is to assume some vaccum expectation value for the 4th component of the gauge field itself:

$$\langle A_4(x, y, t)\rangle \neq 0,$$

(for now on, x stands for the usual spatial coordinates and $x_4 = y$).

Clearly such a statement is not gauge invariant as such, since the value of A_4 at given y can always be rotated away. The corresponding gauge invariant quantity is the line integral of A_4 over a suitable path:

$$\int dl\; A_4.$$

Since we want to keep Lorentz invariance of the remaining $3+1$ dimensions, we will take this path entirely in the y direction and write:

$$X_4 = \int dy\; A_4,$$

assuming X_4 to be time and x independent.

The upper and lower bounds of this integral may vary according to the dimensional reduction scheme: from $-\infty$ to $+\infty$ for non-compactified y (including the case of localisation on a defect), on a circle $[0, 2\pi R]$ for the Kaluza-Klein scheme, on a segment for an orbifold approach. In the case of a closed loop, this is the usual Wilson loop contribution, and can be thought of as the flux of $\overrightarrow{\nabla} \times \overrightarrow{A}$ through the (unphysical) cross section of the torus.

Alternatively [4], in the case of an orbifold, $A_4(y)$ can be gauged away, resulting in an equivalent formulation with non-periodical boundary conditions:

$$\psi'(y) = e^{-i\int_0^y dy A_4(y)}\psi(y).$$

The use of such a line integral to break down symmetry has been developed in details by Hosotani in the framework of dynamical symmetry breaking [5]. We will not discuss here the mechanism for generating such a vaccum expectation value; we turn instead to the physical realisation of CP violation.

In the case of a pure $U(1)$, we have already mentioned that the phase appearing in the mass matrix can be safely rotated away. This was not the case in the model discussed by Thirring. Here indeed, the $U(1)$ Yang-Mills field in fact originates from the $g_{4\mu}$ components of the metric tensor, and inclusion of a torsion term in the coupling to fermions results in a non-minimal coupling:

$$\kappa F^{\mu\nu}\bar{\psi}\sigma_{\mu\nu}\psi,$$

with $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. If $(M + i\gamma_5 X_4)$ and κ are not simultaneously real, rotating the phase in the mass term brings an imaginary component in the magnetic coupling:

$$\kappa' F^{\mu\nu}\bar{\psi}i\sigma_{\mu\nu}\gamma_5\psi,$$

which corresponds in fact to an electric dipole moment, clearly a CP-violating observable.

While we prefer to avoid such non-minimal couplings, a similar situation would occur if we have simultaneously minimally coupled $U(1)$ and $SU(3)$ terms (like in strong interactions); this time the sum of the phase in the mass term and of the θ term (corresponding, in the reduced dimensions, to the anomaly $\theta\,\tilde{G}^{\mu\nu}G_{\mu\nu}$) induces a CP violation.

Of more interest to us however, for later generalisation, is a simple extension based on the $SU(2)$ group that we propose here. Here indeed, neither non-minimal coupling, nor non-perturbative effects are needed.

We start from the Lagrangian:

$$\bar{\Psi}i(\partial^A - iW_a^A\tau^a)\gamma_A\Psi + M\bar{\Psi}\Psi,$$

and assume both $M \neq 0$ and $\langle W_4\rangle = \int dy\, W_4(y) = \begin{pmatrix} w & \\ & -w \end{pmatrix}$. This results in the effective $3+1$ Lagrangian:

$$\begin{pmatrix} \bar{\psi}_1 & \bar{\psi}_2 \end{pmatrix} i(\partial^\mu - iW_a^\mu\tau^a)\gamma_\mu \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} + \begin{pmatrix} \bar{\psi}_1 & \bar{\psi}_2 \end{pmatrix} \mathcal{M} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix},$$

where

$$\mathcal{M} = \begin{pmatrix} M + iw\gamma_5 & \\ & M - iw\gamma_5 \end{pmatrix}.$$

The mass matrix can be diagonalised generally by a bi-unitary transformation, $\mathcal{M}' = U_R^\dagger \mathcal{M} U_L$. In fact, in the present case, the problem is partially undetermined and we can choose ($\alpha = \gamma_5 \arctan\frac{w}{M}$):

$$U_R = \mathbb{I}, \qquad U_L = \begin{pmatrix} e^{-i\alpha} & \\ & e^{i\alpha} \end{pmatrix}.$$

With the fermion masses now diagonal (and degenerate), we obtain two massive $W^\pm$ and one massless W^3 gauge bosons, with the breaking of the gauge symmetry according to the Hosotani mechanism and the effective Lagrangian. The coupling of W^+ and W^- is no longer purely vectorial, but includes a phase between the L and R parts. As a result, a W^3-electric moment is induced at one loop level (see Figure 1 for one example of a contribution).

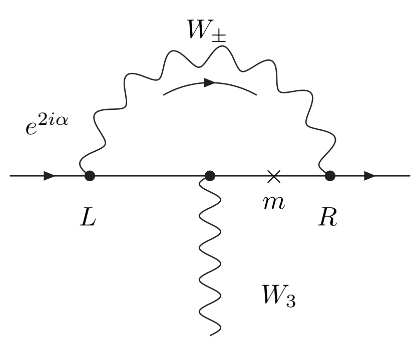

Figure 1:

We find this model interresting for 3 reasons:

- CP violation, dimensional reduction and breaking of the internal symmetry are intimatley linked,
- the approach is purely perturbative,
- the CP vioItation appears in a Kobayashi-Maskawa -like matrix.

Obviously this model has strong limitations, which we list briefly:

- The remaining massless boson cannot be identified with the photon: this can easily be solved by extending to $SU(2) \times U(1)$ (but then the breaking pattern is still a problem, as breaking in the triplet leaves a massless Z boson).
- The coupling of W^+ and W^- is not chiral. While not purely vectorlike in the mass eigenstate basis (L and R have different phases), it is purely vectorlike in the current basis, and certainly does not differentiate significantly between L and R. This is particularly bothersome since the CP generation mechanism appears in the same time here to be directly linked to the presence of both L and R couplings to the gauge bosons.

We will not address the fine details of mass patterns here, and thus will not respond further to the first issue. We deal instead with the second issue in the next section. Let us announce the strategy:

- Obviously in $4+1$ dimensions, vectorlike couplings are automatic due to the Lorentz structure.
- Localisation on a defect (domain wall) or equivalently an orbifold formulation will eliminate either L or R components.
- We choose the defect structure to be part of the internal group; as a result the symmetry group is broken (outside of the defect) and, according to their couplings some L and R components are localised.
- The Hosotani terms then links these remaining L and R components.

Explicit examples based on $SU(3)$ and $SU(4)$ are discussed in the next section.

4 Chiral Examples.

In order to differentiate clearly L and R gauge interactions in the reduced Lagrangian, we thus now consider topological defects which are part of the internal group. As an example, and to be explicit, we will focus on a domain wall scalar field in the adjoint representation.

As it is well known [6], such a defect coupled to fermions localises in its core massless fermionic zero modes with a defined chirality related to the sign of the coupling. For an adjoint scalar coupled to fermions in the fundamental representation, we choose to write the breaking direction as a diagonal operator. As a consequence, the following zero modes are selected :

$$\left(\begin{array}{ccccc} \psi_L^1 & \psi_L^2 & \dots & \psi_R^i & \dots \end{array} \right).$$

Subsequently, as we wish to provide masses to fermions, we have to add scalars which will acquire a constant vev. Those scalars break then the group either inside and outside the wall on the contrary of the domain wall field which breaks the group only outside the wall. Finally, to get complex masses, the Hosotani term has also to be in a non-diagonal direction.

Let now turn to an explicit $SU(3)$ model in order to illustrate the idea. As only 3 chiral localised states arise for the fundamental representation, we expect to form at most one massive and one massless fermion, so even if this example shows both a complex mass term and chiral couplings, CP violation will usually be avoided by rotating away the phase. Yet, this case gives the building principles, we will after that consider an $SU(4)$ which will provide all the desired features.

We start with the following $SU(3)$ invariant fermionic Lagrangian in $4+1$ dimensions:

$$\bar{\Psi} i(\partial^A - iW_a^A \lambda^a)\gamma_A \Psi,$$

to which we first add a domain wall Φ along the 4th coordinates: $m_\Phi \bar{\Psi}\Phi\Psi$.

The choice of Φ in the λ_8 direction of the $SU(3)$ algebra implies thus a confined fermionic representation in the form:[1] $\begin{pmatrix} \psi_L^1 & \psi_L^2 & \psi_R^3 \end{pmatrix}$. Now, we can fill in the second diagonal direction of the group λ_3 with another scalar χ which, when acquiring a vev, will break the group to $U(1) \times U(1)$. As it has already been said, this scalar will not give masses between zero modes, and we must therefore introduce a third scalar H, for instance in the λ_4 direction, which couples ψ_L^1 to ψ_R^3 and breaks the group to $U(1)$. The Hosotani term is forced to be parallel to H in order to obtain the following mass term:

$$\bar{\psi}_L^1 \frac{1}{2}(m_H \langle H \rangle + i\gamma_5 w)\psi_R^3 + h.c..$$

As announced, in this minimal set up, we only generate one fermion mass (whose phase can be removed). Nevertheless, let continue and study the reduced gauge interactions to find out they are indeed chiral.

First of all, a localisation of massless gauge field is also needed. However, since we are mostly interrested in fermions, we will not discuss this issue here and will just mention that it is an open field [7].

Now, taking the localisation of fermions into account, we can see that the only W_1, W_2, W_3 and W_8 gauge bosons are involved in the effective theory, that is in interactions between zero modes. Indeed, the other ones provide interactions between, for instance, a localised left-handed zero mode and an unlocalised left-handed mode with mass of the order of the confining scale. Such interactions therefore do not belong to the effective Lagrangian. Eventually, from the initial gauge interaction, we deduce the effective interactions:

- charged and neutral currents:

$$\frac{1}{\sqrt{2}} W_\mu^+ \bar{\psi}_L^1 \gamma^\mu \psi_L^2, \qquad \frac{1}{2} Z_\mu \bar{\psi}_L^1 \gamma^\mu \psi_L^1, \qquad -\frac{1}{2} Z_\mu \bar{\psi}_R^3 \gamma^\mu \psi_R^3;$$

- and the remaining $U(1)$ current:

$$\frac{-1}{2\sqrt{3}} A_\mu \bar{\psi}_L^1 \gamma^\mu \psi_L^1, \qquad \frac{-1}{2\sqrt{3}} A_\mu \bar{\psi}_R^3 \gamma^\mu \psi_R^3, \qquad \frac{1}{\sqrt{3}} A_\mu \bar{\psi}_L^2 \gamma^\mu \psi_L^2;$$

where we have written the interactions in terms of the mass and $U(1)$ eigenstates. As a result, we get indeed effective gauge interactions which are chiral and possess an electroweak-like structure.

Following the same path, we now turn to $SU(4)$. Let us consider the vaccum configuration:

$$\Phi = \frac{\phi(y)}{2} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \qquad \chi = \frac{\langle \chi \rangle}{2} \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}, \qquad \eta = \frac{\langle \eta \rangle}{2} \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix},$$

with Φ being the domain wall from which the fermion localisation will select an $SU(2)_L \times SU(2)_R \times U(1)_A$ group between zero modes: $\begin{pmatrix} u_L^1 & d_L^1 & u_R^2 & d_R^2 \end{pmatrix}$; while η and χ, acquiring a constant vev, will break down respectively the $SU(2)_L$ and the $SU(2)_R$ subgroups. Those fields fill in all the diagonal space of the algebra. After that, the generation of fermion masses needs non-diagonal scalars H^1 and H^2; e.g.:

$$H^1 = \frac{\langle H^1 \rangle}{2} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \langle H^1 \rangle \lambda_4, \qquad H^2 = \frac{\langle H^2 \rangle}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \langle H^2 \rangle \lambda_{11}.$$

Since those two breaking commute, they clearly minimise their interaction potential, but moreover allow the Hosotani term to get a component in each direction without cost of energy:

$$\int dy\, W^4 = w_4 \lambda_4 + w_{11} \lambda_{11}.$$

This feature provides actually two masses with two phases, i.e.:

$$\bar{u}_L^1 \ \frac{1}{2}(m_1 \langle H^1 \rangle + i w_4 \gamma_5) \ u_R^2 + \bar{d}_L^1 \ \frac{1}{2}(m_2 \langle H^2 \rangle + i w_{11} \gamma_5) \ d_R^2 + h.c..$$

[1] Here, we have to notice that the zero modes are localised differently due to the different strength of the coupling. The localisation of the gauge fields (not discussed here) must take this into account to maintain charge universality.

It can be easily checked that both phases cannot be removed completely from the Lagrangian. Indeed, considering the effective gauge interactions (dropping the 1 and 2 fermionic index):

$$\mathcal{L}_{C.C.} = \frac{1}{\sqrt{2}} W^{+}_{\mu L} \;\; \bar{u}_L \gamma^\mu d_L + \frac{1}{\sqrt{2}} W^{+}_{\mu R} \;\; \bar{u}_R \gamma^\mu d_R + h.c.,$$

$$\mathcal{L}_{N.C.} = \frac{1}{2}(Z_\mu + \frac{A_\mu}{\sqrt{2}}) \;\; \bar{u}_L \gamma^\mu u_L + \frac{1}{2}(-Z_\mu + \frac{A_\mu}{\sqrt{2}}) \;\; \bar{u}_R \gamma^\mu u_R + \frac{1}{2}(Z'_\mu - \frac{A_\mu}{\sqrt{2}}) \;\; \bar{d}_L \gamma^\mu d_L + \frac{1}{2}(-Z'_\mu - \frac{A_\mu}{\sqrt{2}}) \;\; \bar{d}_R \gamma^\mu d_R,$$

in terms of the mass and U(1) eigenstates, we cannot transform the masses to get them real without obtaining a combinaison of the two phases in the charged currents.

The model discussed here is not yet realistic in that charge assignations in the fundamental of $SU(4)$ are not compatible with the observed ones. Also, to have CP violation through the W_L alone, more generations are needed. However, this example shows clearly that CP violation can be generated through dimensional reduction.

For the sake of completeness, we discuss briefly the stability of the potential. Since all the considered scalars are in the adjoint of the group, we can generally take an interaction potential which is minimal when all fields are orthogonal, e.g.: $\rho \; Tr\Phi H + \xi \; (Tr\Phi H)^2$. The Hosotani term implies for the gauged scalars an effective potential from the last component of the covariant derivative, namely: $(Tr[W_4, \Phi])^2$. This contribution is minimal for the two involved fields aligned or commuting together. This term can be used to secure the orientation of one of the scalars parallel to the Hosotani breaking direction. We have constructed explicit examples at the cost of small Yukawa couplings.

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References

[1] W. E. Thirring, Acta Phys. Austriaca Suppl. **9** (1972) 256.

[2] G. C. Branco, A. de Gouvea and M. N. Rebelo, Phys. Lett. B **506** (2001) 115 [arXiv:hep-ph/0012289]. C. S. Huang, T. j. Li, W. Liao and Q. S. Yan, Eur. Phys. J. C **23** (2002) 195 [arXiv:hep-ph/0101002]. D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **87**, 211601 (2001) [arXiv:hep-ph/0103342]. D. Chang, W. Y. Keung and R. N. Mohapatra, Phys. Lett. B **515** (2001) 431 [arXiv:hep-ph/0105177]. Y. Sakamura, Nucl. Phys. B **608** (2001) 279. P. Q. Hung and M. Seco, arXiv:hep-ph/0111013. D. Dooling, D. A. Easson and K. Kang, arXiv:hep-ph/0202206. S. Ichinose, arXiv:hep-th/0206187.

[3] M. Belen Gavela and R. I. Nepomechie, Class. Quant. Grav. **1** (1984) L21.

[4] L. J. Hall, H. Murayama and Y. Nomura, arXiv:hep-th/0107245.

[5] Y. Hosotani, Phys. Lett. B **126** (1983) 309. Y. Hosotani, Phys. Lett. B **129** (1983) 193.

[6] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125** (1983) 136.

[7] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485** (2000) 208 [arXiv:hep-th/0005016]. G. R. Dvali and G. Gabadadze, Phys. Rev. D **63** (2001) 065007 [arXiv:hep-th/0008054]. G. R. Dvali and M. A. Shifman, Phys. Lett. B **396** (1997) 64 [Erratum-ibid. B **407** (1997) 452] [arXiv:hep-th/9612128].